PARADOXES OF SIZE

A TREATISE ON GEOMETRIC VANISHES

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Chapter 1

Introduction

Geometry is the science of describing the world of abstract shapes such as circles, triangles, and so on. Geometry is one of the most logical pursuits. Creating and analysing geometry follows a rigid process that applies the exacting rules of mathematical logic.

The first written accounts of geometry are thousands of years old and can be found on the Rhind and Moscow papyri. The Babylonians used geometry to track the movement of planets across the sky. The Egyptians used geometry to calculate the volume of granaries to feed their population or to construct perfect pyramids to commemorate their god-like Pharaoh. The ancient Greek philosopher Euclid built the foundations of the science of geometry as we know it today (Figure 1.1 on the next page). Even after more than two thousand years, his formulations of geometry are still taught in schools.

Many high-school students fear geometry. They struggle to understand the importance of concepts such as equilateral and isosceles triangles. Geometry describes a world that does not correspond with our direct experience of reality. Many students don't realise that geometry is not just an abstract pursuit. It is the foundation of engineering, which profoundly influences the world we live in. From our houses to our roads and from electronics to navigation, all technology relies on knowledge of geometry.

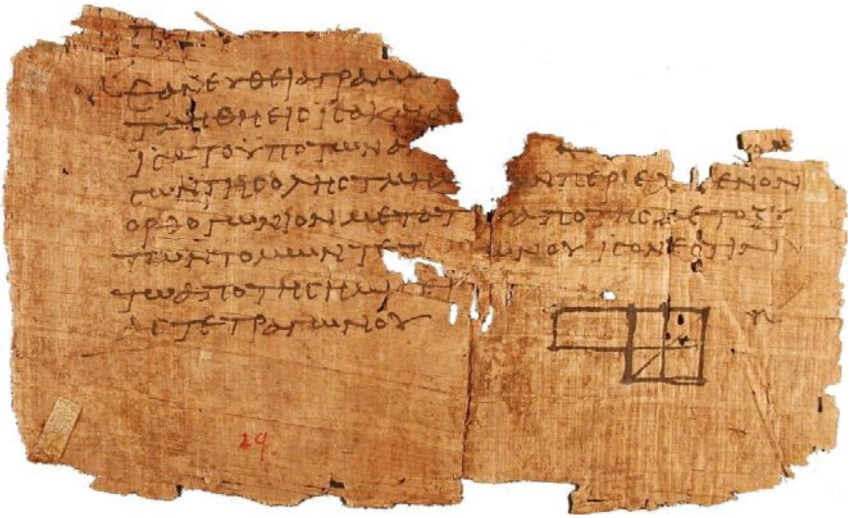

Figure 1.1: Papyrus Oxyrhynchus (page 29). One of the oldest surviving fragments of Euclid's Elements, dated to circa the year 100 (Wikimedia).

At first glance, the science of shapes seems the opposite of magic because of its logical and mathematical nature. This contradiction is, however, superficial. Mathematics and geometry connect our will with the physical world. The craft of geometry describes and changes the physical world. It is in this way that the science of shapes is a form of magic. In another sense, mathematics can seem like magic because mathematical notation has a lot in common with magical incantations.

Geometry is not merely a tool for engineers and scientists. For some people geometry is sacred. Geometrical patterns describe a divine world of order beyond our daily chaotic lives. Geometry becomes sacred and magical when we move beyond mathematics and imbue meaning to the otherwise abstract concepts. The design of almost all sacred buildings around the world follows a geometric regularity, imbued with a sacred meaning. The sacredness of geometry makes it a welcome subject for theatrical magic.

Geometric Vanishes

This ebook discusses geometric vanishes, which are tricks that use geometry to create the illusion that a shape has changed in size. In this magic plot, a geometric shape or image is cut into two or more pieces. When the performer reassembles them, a part of the puzzle or a picture on the puzzle vanishes. Like almost any magic trick, the effect is bidirectional and can also materialise extra bits of the puzzle.

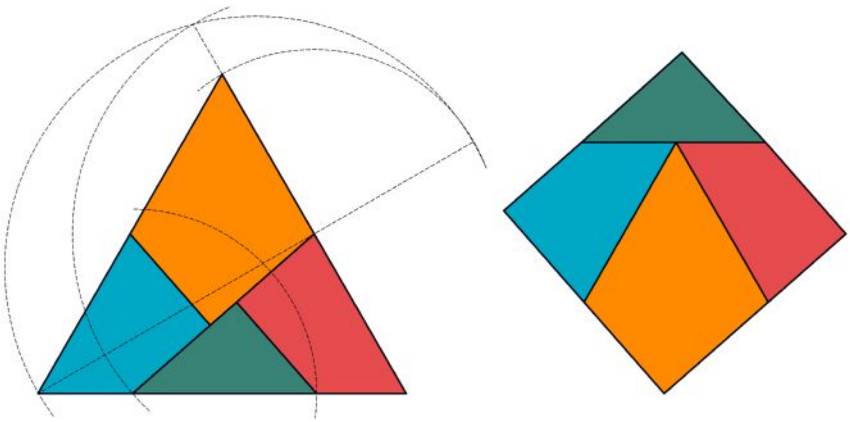

Figure 1.2: Example of a mathematical dissection problem by Henry Dudeney. The Haberdasher's Problem asks to dissect an equilateral triangle using only a compass and straight edge and convert it into a square by rotating the pieces.

The most famous version of this effect is the *Infinite Chocolate Bar*. The original YouTube video was viewed millions of times. In this effect, a bar of chocolate is cut into pieces, reassembled, and an additional piece materialises.

Mathematicians refer to these type of puzzles as dissection problems. In this type of problem, a geometric figure is dissected into several pieces. These pieces are reassembled into a new shape. A mathematician is, for example, interested in how to cut an equilateral triangle with only a compass and straight edge so that the pieces can form a square (Figure 1.2).

Mathematics as a science is the art of defining theorems and proving them. The Wallace–Bolyai–Gerwien theorem states that any polygon can be formed from another by cutting it into a finite number of pieces and recomposing these. This transformation can, of course, only be done if the two polygons have the same area. A geometric vanish is thus, in mathematical terms, an apparent violation of the Wallace-Bolyai-Gerwien theorem.

The name paradox is a deceptive categorisation of this type of magic trick because the change in area is only an illusion. As there is no real paradox, magicians use layers of deception to hide the fact that in reality, nothing has changed.

These effects are possible because of our inability to detect minor changes in size. Martin Gardner calls our inability to judge the size the *principle of concealed distribution*. Human perception cannot perceive minute differences in size, brightness, loudness, mass, length and other stimuli.

Perception psychologists call this Weber–Fechner's law. This rule describes the Just-Noticeable Difference between two objects, excluding visual illusions. In geometric vanishes, the change in size is below the Just-Noticeable Difference, which causes an apparent paradox.

Geometric vanishes as magic tricks

Geometric vanishes look like a typical puzzle problem. While puzzles are amusing, a magic trick should be more than a problem to solve. When geometric vanishes are performed without context or without an additional layer of deception, the performance becomes just a riddle. Max Maven wrote in 1994:

> Magic is not a puzzle, and should not be presented as such . . . However, some have figured out that there can be magic in puzzles.

Geometric vanishes can be quite strong pieces of magic. Some of the YouTube clips with the infinite chocolate trick have millions of views. Even when you know the secret, geometric vanishes remain a fascinating effect.

The deceptive nature of magic tricks requires that the geometrical principles that create the effect remain hidden from the spectator. Many geometric vanishes use additional layers of deception to deflect attention away from the mathematical solution. Meaningful presentations, occasional sleight of hand or gimmicks can enhance the level of astonishment.

This book discusses three types of dissection fallacies. Each of these uses variations of the principle of concealed distribution. The vertical line paradox is the principle behind the famous *Vanishing Leprechaun*. The second type is the *Curry Paradox*, which relies on properties of the Fibonacci sequence. Tangram is a well-known Chinese puzzle that also hides an apparent paradox. The *Tangram Paradox* is the third and most popular form of geometric vanish.

Figure 1.3: Conundrum *by Bill Montana and Dr Paul.*

The following three chapters discuss the principles and the performance history of each of these three types of geometric vanishes. The story of geometric vanishes shows an evolution from a mathematical error to amusement, and from an apparent paradox to a performance piece for magicians. But before we delve into the world of geometric vanishes as theatrical magic, we briefly explore the prehistory of these effects through the work of the great puzzle maker Sam Loyd.

Chapter 6

Epilogue

This excursion into the geometric vanish plot in magic reveals an evolution from a mathematical curiosity towards a deceptive magic trick. Magicians improved this primordial version of this magic trick by adding additional layers of deception.

The development of this trick involves the work of several people over almost half a millennium. The geometric vanish started with Sebastiano Serlio, who made a math mistake when he described how to partition a tabletop. In the 18^{th} century, William Hooper and Edmé-Gilles Guoyot developed Serlio's miscalculation into a puzzle in their books about amusing sciences.

Victor Schlegel further investigated the mathematics of this curiosity in the 19^{th} century, and he found that using the Fibonacci numbers provides the most deceptive solution. Walter Dexter developed an alternative arrangement of the Fibonacci bamboozlement a few decades later.

The legendary puzzle maker Sam Loyd brought the puzzle into the 20^{th} century and further developed the plot of the geometric vanish into two distinct effects. In the basic version, the area of the puzzle changes. In the new version, the area of the puzzle remains the same, but a part of the drawing vanishes.

These principles came to the attention of magicians through the work of Paul Curry and Martin Gardner. Paul Curry developed several versions of the paradox that would eventually carry his name. The figurative version became very popular with the release of the *Vanishing Leprechaun* in the late 1960s. The other type of geometric vanishes required some more development before magicians would start using it in their act.

Winston Freer and Mitsonobu Matsuyama improved this magic plot by applying a new principle. They used the Tangram paradox to change the area of a tiling puzzle magically. This method is more deceiving because there are no small gaps that explain the missing space. Matsuyama's version also removed the need to count squares so that the geometric vanish became a proper performance piece.

The latest versions in this magic plot no longer rely on mathematics but use sleight-of-hand or gimmicks to create the illusion of magic. With these developments, the geometric vanish has become a mature plot in magic performances.

Geometric vanishes are an example of how magic often originates from a mistake. Magicians view the world differently to other people. For a magician, any event with an unexpected outcome can be the starting point for a new magic trick. Geometric vanishes are just one of many examples that show how magicians innovate to maximise the deceptive qualities of simple principles to further their art.

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